Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS

2A/2B

Calculator-assumed

WACE Examination 2010

Final Marking Key

This 'stand alone' version of the WACE Examination 2010 Final Marking Key is provided on an interim basis.

The Standards Guide for this examination will include the examination questions, marking key, question statistics and annotated candidate responses. When the Standards Guide is published, this document will be removed from the website.

Question 7 **(8 marks)**

Brian designs a three-character password made from the characters A, B, # and 1, without using a character more than once in the password. There are 24 different possible passwords which are listed below.

A B #	B A #	# A B	1 A B
A B 1	B A 1	# A 1	1 A #
A # B	B # A	# B A	1 B A
A # 1	B # 1	# B 1	1 B #
A 1 B	B 1 A	# 1 A	1 # A
A 1 #	B 1 #	# 1 B	1 # B

(a) Brian states that the probability that his password will end in the # character is 0.4. Is Brian correct? Justify your answer. (2 marks)

Solution
$\text{P(ends \#)} = \frac{6}{24} = 0.25$ $0.25 \neq 0.4$ Therefore Brian is incorrect
Specific Behaviours
✓ correctly calculates P(ends #) ✓ makes conclusion based on values

(b) Using the list above, determine the probability that Brian's password

(i) contains the # character. (1 mark)

Solution
$\frac{18}{24}$
Specific Behaviours
✓ correctly calculates P(contains #)

(ii) ends in the letter A. (1 mark)

Solution
$\frac{6}{24}$
Specific Behaviours
✓ correctly calculates P(ends A)

(iii) starts with the number 1 **or** ends with the # character. (1 mark)

Solution
$\frac{10}{24}$
Specific Behaviours
✓ correctly calculates P(start 1 or end #)

(c) Ava decides to make her own three-character password from the characters %, V and 5, without using a character more than once in the password.

(i) List all of the possible passwords Ava could make. (2 marks)

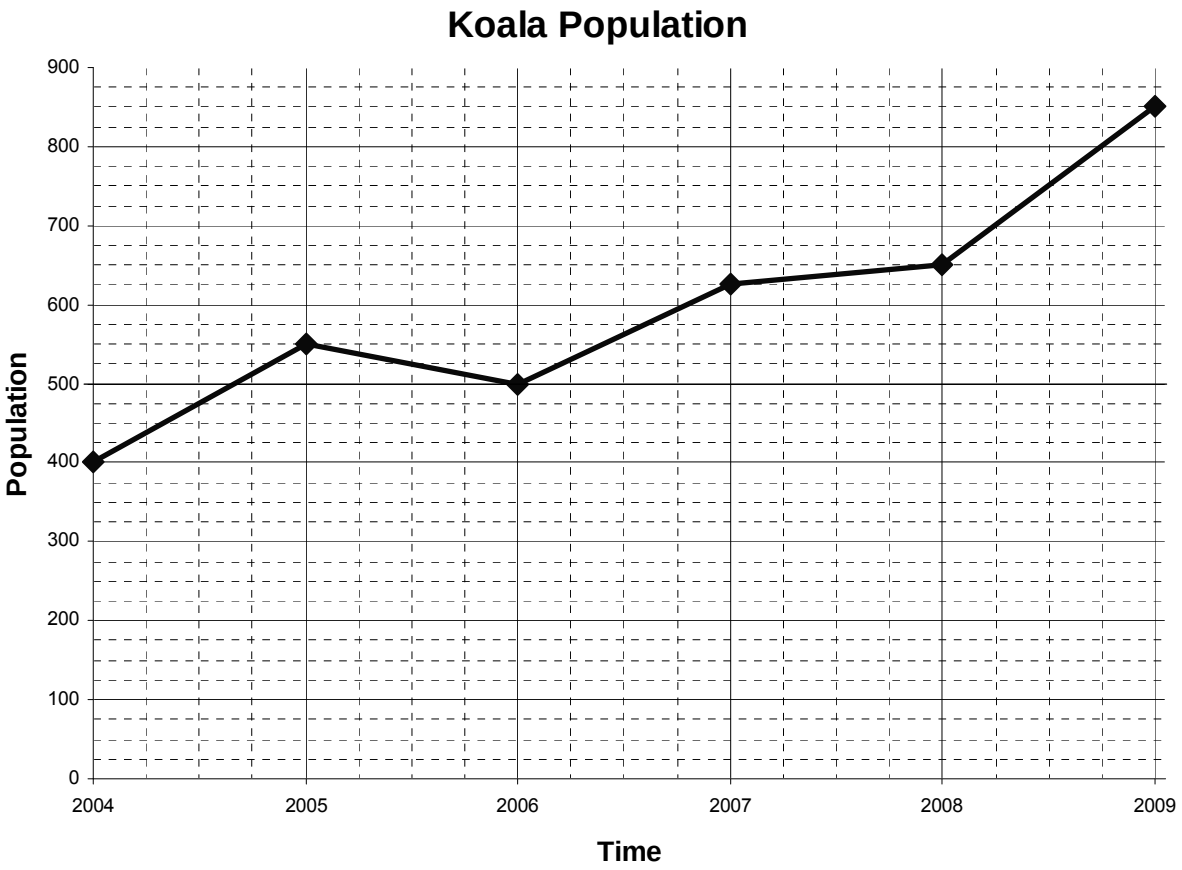
Solution					
%, V, 5,	%, 5, V	V, %, 5	V, 5, %	5, V, %	5, %, V
Specific Behaviours					
✓ lists at least 4 possibilities ✓ lists every possibility					

(ii) Brian tries to guess Ava's password. What is the probability that Brian guesses Ava's password correctly with only one guess? (1 mark)

Solution
$\frac{1}{6}$
Specific Behaviours
✓ correctly calculates probability

Question 8 **(7 marks)**

(a) The population of koalas in a national park is recorded at the start of every year. The data collected from 2004–2009 are plotted below.



(i) Describe the trend in the population of koalas against time. (1 mark)

Solution
increasing
Specific Behaviours
✓ correctly identifies trend

(ii) What assumption has been made by the joining of the points in the graph above? (1 mark)

Solution
The population steadily increases or decreases (at the same rate during each year)
Specific Behaviours
✓ identifies a constant rate of change, or increasing (at the same rate) uniformly.

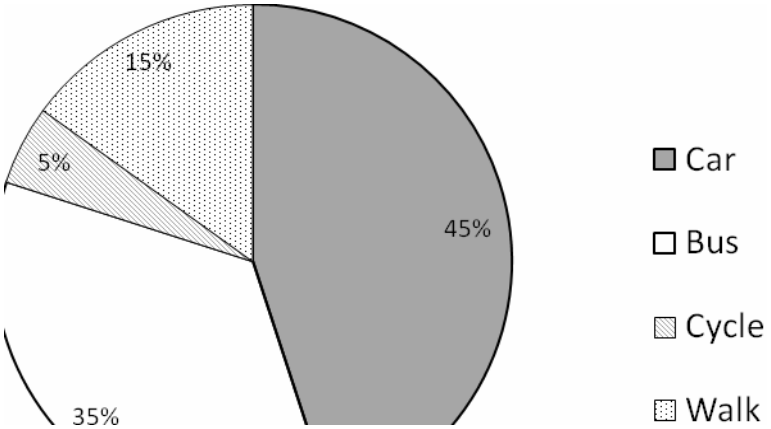
(iii) Estimate the population half-way through the year 2005. (1 mark)

Solution
525
Specific Behaviours
✓ correctly reads value from graph between calibrations

(iv) Approximately when did the population first exceed 700? Give your answer as a month and year, e.g. October 2005. (1 mark)

Solution
April 2008 (accept March 2008)
Specific Behaviours
✓ correctly gives month and year

(b) Annie and Jimmy surveyed a group of teenagers to find out how they travelled to their part-time jobs one Saturday. They drew a pie chart to show the results.



(i) If 14 teenagers caught buses to their jobs on the Saturday, how many teenagers did Annie and Jimmy survey? (2 marks)

Solution
$\frac{35}{100} \times N = 14$ $N = 40$
Specific Behaviours
✓ writes an equation to solve that will give the correct value of N or ✓✓ correctly determines the number of teenagers

(ii) A teenager is selected at random from the group surveyed. What is the probability that they walked or cycled to their job on the Saturday? (1 mark)

Solution
20% or 0.2 or 1/5
Specific Behaviours
✓ correctly calculates probability

Question 9 **(5 marks)**

(a) Consider the sequence of numbers below.

24, 60, 150, 375, …

(i) State a recursive rule in words for this sequence. (1 mark)

Solution
The next term is two and half (2.5) times the previous term
Specific Behaviours
✓ states the link between consecutive terms

(ii) Write a recursive rule for this sequence using algebraic notation. (2 marks)

Solution
$\frac{60}{24}=\frac{150}{60}=2.5$ $T_{n+1}=2.5\times T_n$ $T_1=24$
Specific Behaviours
✓ correctly writes the recursive equation ✓ states the value of the first term

(b) The sequence of numbers: 12, 67, 342, 1717 … can be defined recursively as:

$$T_{n+1}=5T_n+k,\quad T_1=12$$

(i) Determine the value of k. (1 mark)

Solution
$T_2=5\times T_1+k$ $67=5(12)+k$ $k=7$
Specific Behaviours
✓ correctly calculates value of k

(ii) Determine the value of n when this sequence first exceeds one million. (1 mark)

Solution
$n=8$
Specific Behaviours
✓ correctly determines the value of n based on (i)

Question 10 **(6 marks)**

(a) When Granny cooks some rice, she adds 4 cups of water to 3 cups of rice.

How many cups of water does she need for 5 cups of rice? Give your answer as a mixed numeral, e.g. $6\frac{1}{2}$ cups. (2 marks)

Solution
$4:3$ $x:5$ $\frac{x}{4}=\frac{5}{3}$ $x=6\frac{2}{3}$
Specific Behaviours
✓ writes the correct ratios ✓ solves for the unknown expressing the answer as a mixed numeral

(b) A pack of 64 Ymiracle nappies costs \$21.95. A bulk package of Peonies containing 144 nappies costs \$52.49.

Which brand is the better buy? Justify your answer. (2 marks)

Solution
Ymiracle $21.95\div 64=0.34$ (2 decimal places) Peonies $52.49\div 144=0.36$ (2 decimal places) Ymiracle is cheaper
Specific Behaviours
✓ calculates unit prices for both brands ✓ correctly states the cheaper brand based on comparison

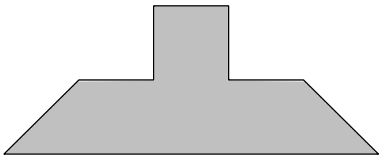
(c) Margaret runs a boutique. She makes a profit of between 10% and 40% on the cost price of all clothes sold.

Margaret sells a dress for \$280 with the profit in her desired range. Determine the lowest and highest price Margaret could have paid for the dress. (2 marks)

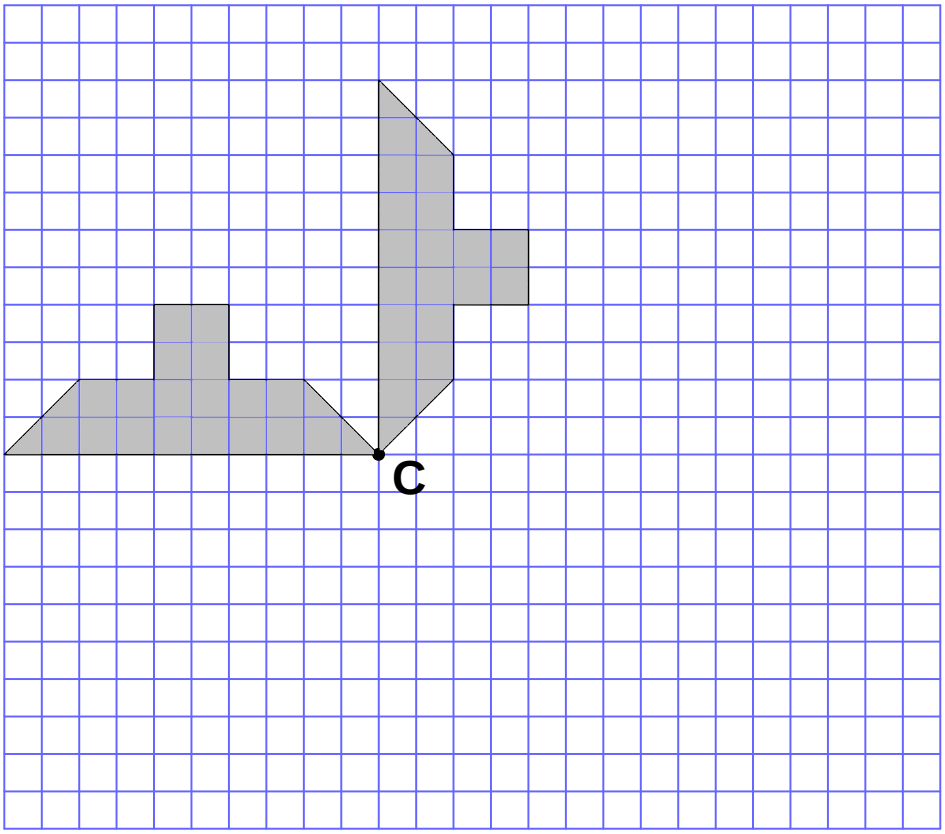
Solution	
$x\times 1.1=280$	$y\times 1.4=280$
$x=254.55$	$y=200$
Dress bought between \$200 and \$254.55	
Specific Behaviours	
✓ calculates the lower price ✓ calculates the upper price	

Question 11 **(6 marks)**

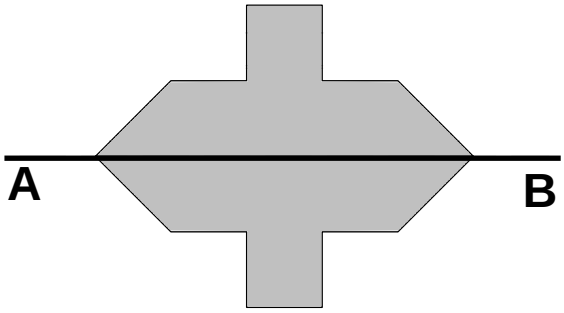
Consider the shaded shape drawn below.



(a) Rotate the shaded shape 90^{o} clockwise about the point C. (2 marks)

Solution

Specific Behaviours
✓ rotates the object 90 degrees ✓ rotates the object clockwise about C

(b) The diagram below shows the original shape reflected about the line AB. Consider the original shape and image as one combined object.



(i) How many lines of symmetry does this combined object have? (1 mark)

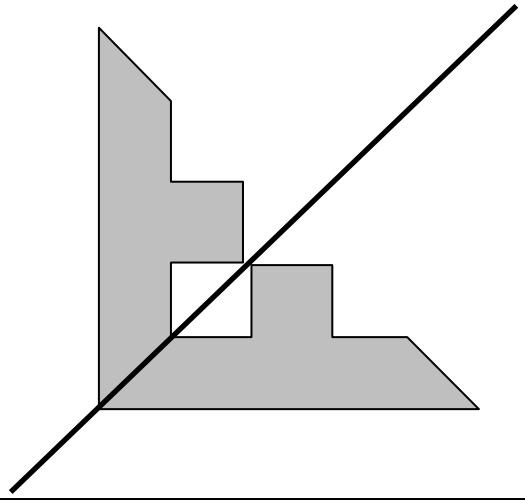
Solution
2
Specific Behaviours
✓ correctly states the number of lines of symmetry

(ii) What is the order of rotational symmetry for this combined object? (1 mark)

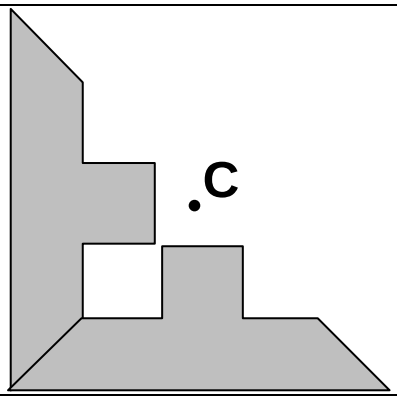
Solution
2
Specific Behaviours
✓ correctly states the order of rotational symmetry

(c) A new combined object (shown below) could be produced either by a reflection or a rotation of the original shape.

(i) If the transformation was a reflection, draw in the line that the original shape was reflected about. (1 mark)

Solution

Specific Behaviours
✓ correctly draws in line

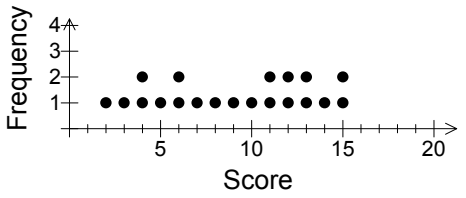
(ii) If the transformation was a rotation, mark the point C where the original object was rotated about. (1 mark)

Solution

Specific Behaviours
✓ correctly places dot

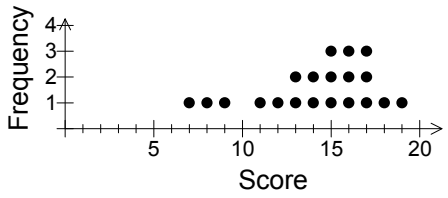
Question 12 **(10 marks)**

Moya the mathematics teacher wanted to check whether the students in her class had improved their understanding of chance. She gave the students a pre-test (out of 20) at the beginning of the topic and a post-test (out of 20) at the end. The score for each student on each test is displayed in the dot frequency graphs below.

Pre-test



Post-test



(a) The teacher commented: 'Looking at the dot frequency graphs, it is clear that everyone improved from the pre-test'. Why is this statement false? (1 mark)

Solution
From the dot plots we do not know which student scored what result. For example, a student may have scored 10 in the pre-test but 8 in the post-test.
Specific Behaviours
✓ identifies that we do not have individuals scores for both tests

(b) Some summary statistics have been provided. Complete the table by calculating the remaining summary statistics. (3 marks)

Solution

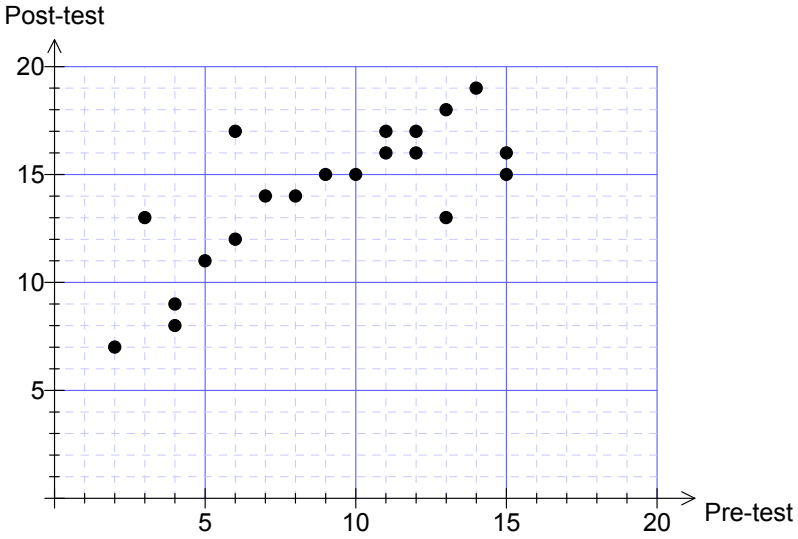
Statistic	**Mean**	**Median**	**Range**
Pre-test	**9**	**9.5**	13
Post-test	14.1	15	**12**

Specific Behaviours
✓ correctly states mean for the pre-test ✓ correctly states median for the pre-test ✓ correctly states range for the post-test

(c) Compare the two data sets, commenting on the distribution of the graphed data. Has the class as a whole improved from the pre-test to the post-test? (2 marks)

Solution
Pre-test data set was more uniformly distributed whereas post-test is more symmetrical (though skewed) As a class the results have improved as the mean and median have both increased by about 5.
Specific Behaviours
✓ compares the shape of the dot plots ✓ justifies an improvement by looking at mean and/or median (not just range)

The test results were plotted on the scatterplot as shown below.

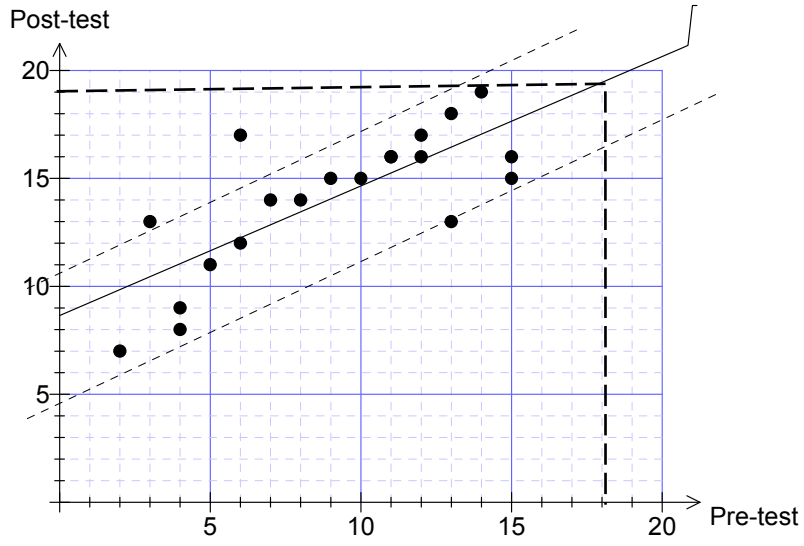


Use the scatterplot and any previous information to answer the following:

(d) What result did the student who scored 12 in the post-test achieve in the pre-test?
(1 mark)

Solution
6
Specific Behaviours
✓ identifies pre-test score from scatterplot

(e) Draw in a trend line on the scatterplot above. (1 mark)

Solution

Specific Behaviours
✓ draws a trend line by eye which is within the bounds of the dotted lines

(f) Show how to use your trend line to predict a post-test score for a student who scored 18 in the pre-test but was absent for the post-test. Comment on the reliability of your prediction. (2 marks)

Solution
See diagram (e) Post-test approx 19/20 Unreliable due to extrapolation
Specific Behaviours
✓ check student answer from diagram ✓ comment on unreliability of prediction because of extrapolation

Question 13 **(6 marks)**

At the start of spring, Jennifer planted 120 tulip bulbs. She watered them regularly and after three weeks measured their height. The results are displayed in the frequency table below.

Height of tulip, h (cm)	Frequency
$0 \leq h < 5$	2
$5 \leq h < 10$	0
$10 \leq h < 15$	0
$15 \leq h < 20$	0
$20 \leq h < 25$	12
$25 \leq h < 30$	27
$30 \leq h < 35$	34
$35 \leq h < 40$	21
$40 \leq h < 45$	12
$45 \leq h < 50$	5
$50 \leq h < 55$	7

(a) Determine the mean height of the tulips, correct to one decimal place. (2 marks)

Solution
mean = 33.5 cm
Specific Behaviours
✓ calculates mean ✓ correctly rounds to one decimal place

(b) Determine the class interval which contains the median. (1 mark)

Solution
$30 \leq h < 35$
Specific Behaviours
✓ determines correct interval

(c) Determine the proportion of tulips that were less than 30 cm high. (1 mark)

Solution
$\frac{41}{120} = 0.34\ (2d.p)$
Specific Behaviours
✓ determines correct proportion.

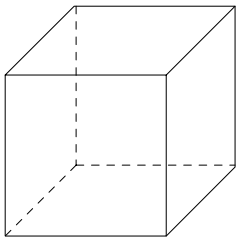
(d) Jennifer wants to display some of her tulips at the annual spring fair. To be eligible, tulips must be at least 30 cm high. What is the highest percentage of Jennifer's tulips that could be displayed at the fair? (2 marks)

Solution
$\frac{79}{120} \times 100 = 65.8\%$
Specific Behaviours
✓ determines number of tulips that could be displayed

✓ expresses this number as a percentage

Question 14 **(4 marks)**

Consider the solid cube shown below.

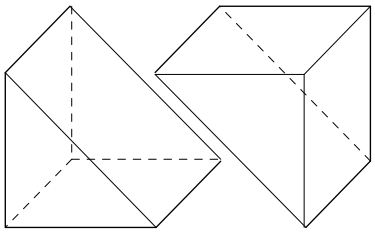


10 cm

(a) Determine the total surface area of this cube. (1 mark)

Solution
$SA = 6 \times 10^2$ $= 600 \text{ cm}^2$
Specific Behaviours
✓ correctly calculates the surface area

This solid cube is to be cut into two equal pieces, as shown in the diagram below.



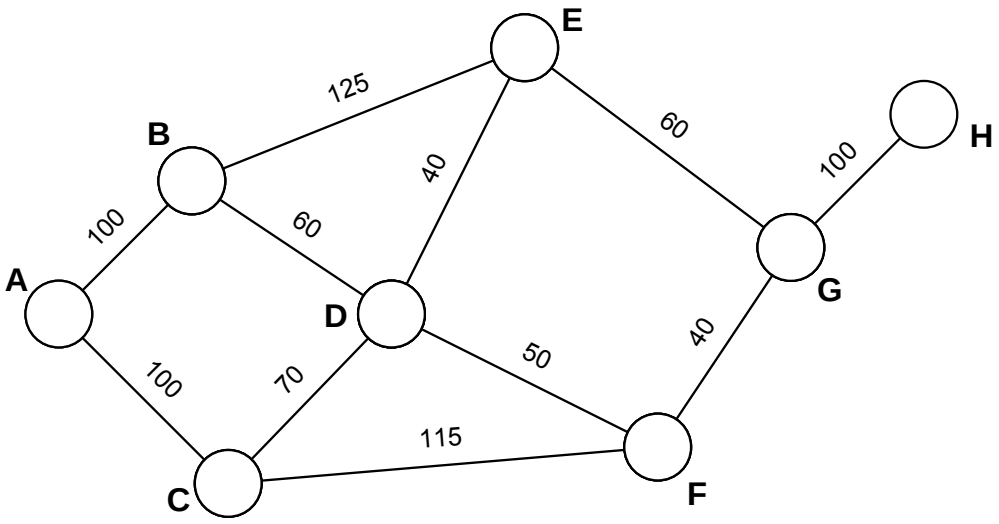
(b) What difference in total surface area does this cut make when compared to the total surface area of the cube obtained in (a)? Show working to justify your answer.

(3 marks)

Solution
$SA = 6 \times 10^2 + 2 \times 10 \times \sqrt{10^2 + 10^2}$ $= 882.8 \text{ cm}^2$ $\therefore$ SA increases by 282.8 cm^2 **or** $\text{Diff} = 2 \times 10 \times \sqrt{10^2 + 10^2}$ $= 2 \times 14 \cdot 14 \times 10$ $= 282.8 \text{ cm}^2$
Specific Behaviours
✓ determines length of cut side ✓ correctly determines the area of the new rectangular face ✓ correctly determines the difference the cut makes to the SA

Question 15 **(5 marks)**

The network below shows the distances (metres) and connections between a series of classroom blocks, A to H, at a local high school.



(a) Is the network of classroom blocks traversable? Explain why/why not. (1 mark)

Solution
No because it has 6 odd vertices. To be traversable it needs 0 or 2 odd vertices
Specific Behaviours
✓ refers to the number of odd vertices as a reason why it is not traversable

(b) Determine the shortest way to travel from Block A to Block H and state this shortest distance. (2 marks)

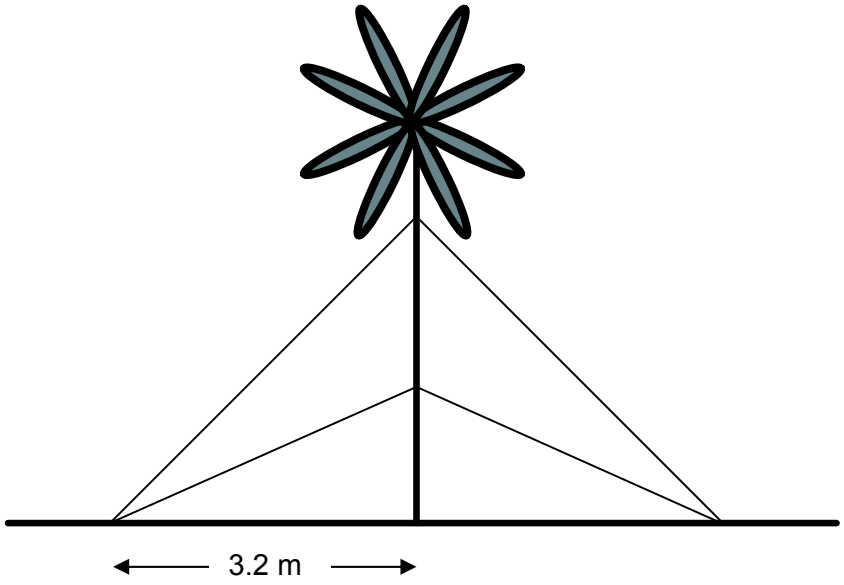
Solution
ABDFGH, 350 metres
Specific Behaviours
✓ states correct path ✓ states correct distance

(c) Explain how, if at all, the shortest path from Block A to Block H is affected if you must travel via Block C. (2 marks)

Solution
The shortest distance is now 355 metres, path is ACFGH Increased distances by 5 metres
Specific Behaviours
✓ uses change to determine the shortest path ✓ describes effect of change

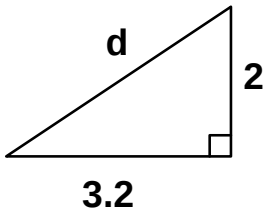
Question 16 **(6 marks)**

A palm tree has been moved from a plant nursery to a private garden in the north of Western Australia. The tree must stand vertically on horizontal ground, so it is supported by a number of wires. All of these wires have one end attached to the ground 3.2 metres from the base of the tree. The other ends are attached to points that are either one-third or two-thirds of the way up the trunk of the tree.



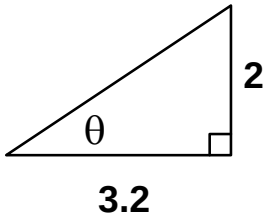
After planting, the trunk height of the palm is 6 metres.

(a) Determine the length of one of the short wires, to the nearest 10 cm. (2 marks)

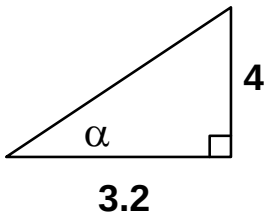
Solution
 $d = \sqrt{2^2 + 3.2^2}$ $= 3.77 \text{ m}$ $= 3.8 \text{ m}$ **or** 380 cm
Specific Behaviours

✓ calculates length
✓ correctly rounds to the nearest 10 cm

(b) Use trigonometry to determine the angle a short wire makes with the ground, to the nearest degree. (2 marks)

Solution
 $\tan(\theta) = \frac{2}{3.2}$ $\theta = 32.01°$ $\theta = 32°$
Specific Behaviours
✓ uses tangent ratio correctly ✓ correctly calculates angle size

(c) Determine the angle between a short wire and a long wire, to the nearest degree. (2 marks)

Solution
 $\tan(\alpha) = \frac{4}{3.2}$ $\alpha = 51.3°$ $\alpha = 51°$ Angle between long and short wires $= 51 - 32$ $= 19°$
Specific Behaviours
✓ calculates size of angle alpha ✓ correctly calculates angle size to nearest degree

Question 17 **(6 marks)**

Sisters Breanna and Stephanie are each given $40 to spend at the second-hand book fair. All non-fiction books are sold at one fixed price and all fiction books are sold at a different fixed price. Breanna buys eight non-fiction and five fiction books and is given $1 change. Stephanie buys four non-fiction and ten fiction books and receives $4 change.

(a) Let n be the price of a non-fiction book and f the price of a fiction book. Write two linear equations from the above information. (2 marks)

Solution
Breanna: $8n+5f=39$
Stephanie: $4n+10f=36$
Specific Behaviours
✓ correctly determines total amounts, i.e. $39 and $36
✓ correctly expresses items bought by Breanna and Stephanie

(b) Determine algebraically the cost of buying a non-fiction book. Give your answer in dollars and cents. (2 marks)

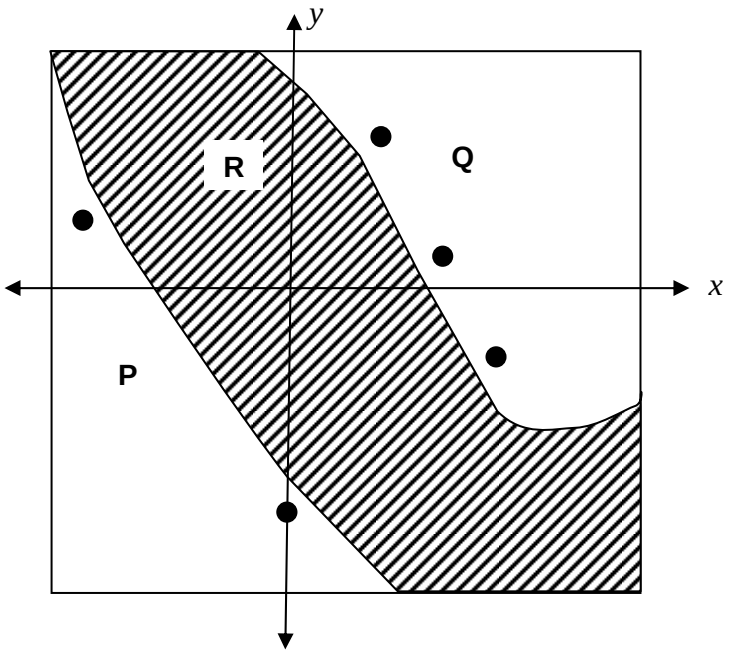
Solution
$16n+10f=78$
$4n+10f+36$
i.e. $12n=42$
$n=3.5$
∴ non-fiction books cost $3.50
Specific Behaviours
✓ simplifies equations to one variable
✓ gives correct price in dollars and cents (not n = 3.5)

(c) Their little sister Alicia is only given $20 to spend. She wants to buy five fiction and two non-fiction books. Does she have enough money? Justify your answer. (2 marks)

Solution
$n=3.5$
$f=2.2$
Alicia spends $=2\times3.5+5\times2.2$
$=\$18$
Alicia has enough money by $2
Specific Behaviours
✓ correctly calculates money Alicia needs
✓ states Alicia has enough money

Question 18 **(6 marks)**

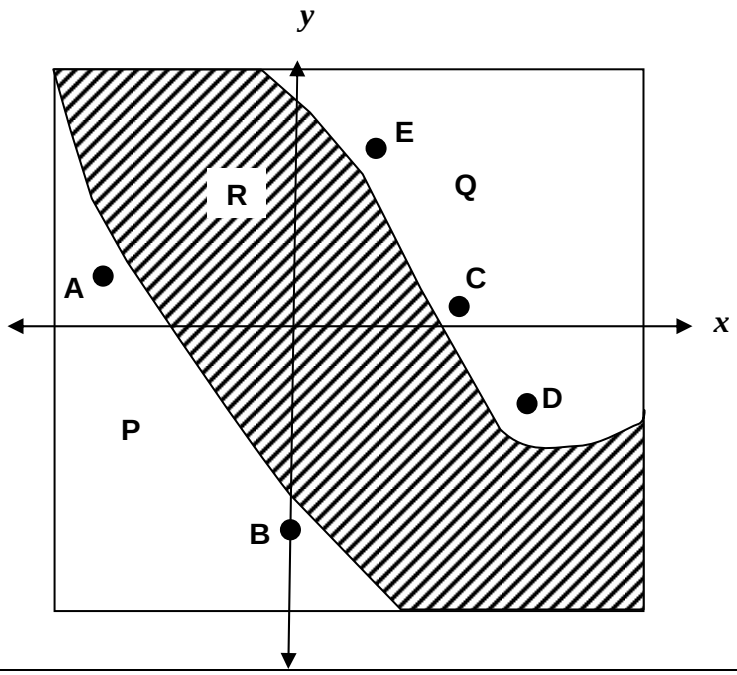
A bridge is to be built to connect land P and land Q, which are separated by river R as shown in the diagram below (not drawn to scale).



The government wants to minimise the cost of building the bridge, so it is in favour of building the shortest bridge possible. Five points have been chosen: A(–5, 2), B(0, –6), C(4, 1), D(6, –4) and E(2, 5).

(a) Correctly label each of the points A, B, C, D and E on the diagram above. (2 marks)

Solution



Specific Behaviours

✓ correctly labels three points
✓ correctly labels all points

(b) It has already been determined that a bridge from A to D would be 1250 metres long, while a bridge from B to E would be 1120 metres long. Given that 1 unit is 100 metres, decide which bridge should be built and how long (to the nearest 10 metres) it would be. Justify your answer. (4 marks)

Solution
$AE = \sqrt{7^2 + 3^2} = 7.62$ $AC = \sqrt{9^2 + 1^2} = 9.06$ $BC = \sqrt{4^2 + 7^2} = 8.06$ $BD = \sqrt{6^2 + 2^2} = 6.32$ BD is shortest with a length of 630 m
Specific Behaviours
✓ correctly determines one length ✓ correctly determines other lengths ✓ states BD is the shortest bridge ✓ rounds BD correctly to the nearest 10 metres

Question 19 **(5 marks)**

Consider the first four rows given below.

(a) Extend the pattern below for rows 5 and 6. (2 marks)

Solution			
Question		**Result**	
$1\times 2+11$	=	13	row 1
$2\times 3+11$	=	17	row 2
$3\times 4+11$	=	23	row 3
$4\times 5+11$	=	31	row 4
$\mathbf{5\times 6+11}$	=	**41**	row 5
$\mathbf{6\times 7+11}$	=	**53**	row 6

Specific Behaviours
✓ correctly completes row 5 ✓ correctly completes row 6

(b) Determine the result for row 25. (1 mark)

Solution
$25\times 26+11=661$
Specific Behaviours
✓ correctly calculates the result for row 25

(c) Con looked at the above results and conjectured that every result is a prime number (a number with only two factors, 1 and itself). Test three further cases and comment on whether they support or refute his conjecture. (2 marks)

Solution
$10\times 11+11=121$ (one possible answer) $121=11\times 11$ therefore 121 is not prime
Specific Behaviours
✓uses patterns correctly to test further cases ✓comments on conjecture – conjecture is refuted because counter example is found or conjecture is supported by the three test cases